

A Mathematical Model to Determine the Electrical Energy Production in Photovoltaic Fields Under Mismatch Effect

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Abstract -- Among the electrical energy production systems from renewable sources, the photovoltaic one represents today one of the most efficient and technically tested choice. In this paper the Authors present a mathematical model that allows the maximization of the generated power of photovoltaic (PV) plants under unequally radiation conditions.

In particular, thanks to the model of a PV plant, unequally radiated, developed in Matlab-Simulink environment, in this paper the mismatch problem of PV plants is dealt with. The simulations carried out allow to assert that by putting into action re-configuration techniques of series-parallel connections of PV plants, under unequally radiation, can allow to optimize the electrical energy production from these plants.

This result opens scenarios of conceivable modifications to the PV field configurations today chosen during the design stage and unchangeable during the operating stage.

Index Terms -- Photovoltaic cells, Solar power generation, Mismatch effect

I. INTRODUCTION

In the last years, the constant grow of the renewable electrical energy plants in electrical systems is due to the liberalization processes of the electrical energy European market and to the international engagement of Europe for the reduction of carbon gas emissions.

In this frame, photovoltaic generation systems have the opportunity to be as much as suitable for their important advantage being able to produce electrical energy very close to the electric loads. In this way the transmission losses are avoided and it is also possible to satisfy the daily load diagrams' peaks since they supply the maximum power quite in correspondence to the maximum request.

The photovoltaic plants, moreover, do not emit pollutant emissions, do not vibrate and, thanks to their modularity, can comply with the morphology of the installation sites and so they present a lower environmental impact with respect to other renewable energy systems.

Against these vantages, although Italy is in a favourable climatic condition, since its ground is characterized by high values of the annual average insolation, the kWh cost from photovoltaic source is high

with respect to the one from other renewable resource like the wind.

In this work, some techniques which allow the optimization of the production from PV plants under unequally radiation conditions are examined.

The PV plants are today built in a fixed series-parallel configuration and the single module is equipped with a bypass diode. This is used to bypass the single module when it is slightly radiated in order to avoid that the single module current may reduce the current of the whole photovoltaic string.

On one hand this solution is easily adoptable and allow to improve the energy production from the whole PV string but on the other hand it imposes to renounce to the energy produced by the module which is bypassed by the diode.

In the grid-connected plant the eventuality of the unequally radiation of the modules can be limited thanks to designing solutions and thanks to the periodical maintenance which removes bounded shadings (plastic bags or excrements on the modules). Different is for plants feeding cars which must work under unequally radiation conditions.

II. THE MISMATCH EFFECT

The mismatch term indicates the electrical maladjustment among the photovoltaic (PV) cells of the entire module.

The causes of this maladjustment are attributable to factors which are internal or external to the single cell.

Among the first ones, it is included: the not homogeneous external characteristics of the cells, due to dissymmetric manufacturing, the degradation of the cell blooming layer, the manufacturing defects, the possible breaking of the cells; among the second ones it is included: the dirt on the anterior part of the cells, the degradation of the materials used for the cells encapsulating and the unequally radiation of the cells.

All these factors lead to a reduction of the module performances implying that the generated power of a module is less than the sum of the generated power of the single cells.

The improvement of the modules production could be reached thanks to an appropriate choice of the cells implying however an increase of the modules

manufacturing costs. For this reason, it is sometimes preferable to accept a generated power reduction even equal till the 5% of the power derivable from all equal cells.

The partial shading of one cell or the unequally shading of a group of cells could be caused by clouds or by a layer of smog unequally spread limiting so the electrical power produced by the PV plants. This can cause the heating of a restricted part of the cell (hot spot phenomenon) till its possible breaking.

In order to avoid these drawbacks, a bypass diode is inserted in parallel to a group of cells that constitute a solution to the problem of the modules shading has to be taken into account during the design stage by searching some plant configurations which allow the best performances achievement. Moreover, it is important to evaluate, according to the installation site, the best disposition of the panels which may reduce, during the day, shading of restricted parts of the modules.

It is also necessary to evaluate how the solar radiation changes during the day and the year in order to choose the best position and inclination of the panels.

As regards the plant the traditional configuration with the whole plant connected to one single inverter (central inverter) is substituted by the string technology according to which each PV string is connected to one inverter.

For each inverter about 20/30 modules are expected and the number of inverters is chosen on the basis of the total required electrical power. The string technology, with reference to the shading problem, improves the PV system performances compared with the centralized architecture.

Interesting results could be reached thanks to the introduction of reconfigurable plants. Today plants, in fact, are in a fixed configuration.

In the past, some surveys on re-configurable plants had been carried out not with the aim of obtaining voltage and current output levels suitable to the need of the variable loads.

As shown in this work, the adoption of suitable re-configuration techniques may maximize the production of electrical energy under unequally radiation conditions.

In this context, it is important to be able to use mathematical models useful to simulate PV plants under unequally radiation conditions.

III. MATHEMATICAL MODEL OF ONE PV MODULE

In scientific literature many mathematical models which allow to simulate the behaviour of a PV module exist [2 ÷ 6].

Several studies have been carried out starting from models with distributed parameters to get to the conclusion that the four parameters model represents an optimum compromise between the calculation burden and the approximation of the modelling [2].

The four parameters model represents one irradiated PV module like a current generator in parallel with one diode.

The value of the generator current I_L is function of the number of over-carriers caused by the solar radiation;

the Shockley equation explains the current circulating into the diode.

The model, takes into account the variation of the photoelectric current when the radiation and the temperature change, and also the variation of the diode saturation current when the temperature changes.

The electric circuit of the mathematical model of the module taken into account is reported in Fig. 1. In this, the current generator I_L represents the generated photoelectric current, while the diode and the resistance R_s , which takes into account the internal electrical losses, model the photovoltaic module.

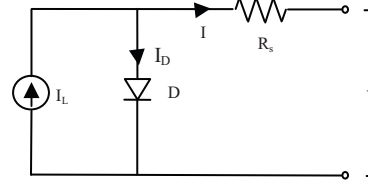


Fig. 1 Equivalent electric circuit of a PV module

By applying the Kirchoff law to the node of the circuit reported in Fig. 1 the current I produced by the photovoltaic module is obtained.

$$I = I_L - I_D \quad (1)$$

where:

- I_D is the diode current;

- I_L is the photoelectric current related to a given condition of radiation and of temperature.

The I_D diode current is given by the Shockley equation:

$$I_D = I_o \left[\exp \left(\frac{q(V + IR_s)}{\gamma k T_C} \right) - 1 \right] \quad (2)$$

where:

- V is the output voltage [V];

- I_o is the saturation diode current [A];

- γ is the form factor which represents an index of the cell failing;

- R_s is the series resistance of the cell [Ω];

- q is the electron charge (1.602×10^{-19} C);

- k is the Boltzmann constant (1.381×10^{-23} J/K);

- T_C is the module temperature [K]

By substituting (2) into (1) the following equation is obtained which represents the I-V module characteristic curve under generic radiation and temperature conditions.

$$I = I_L - I_o \left[\exp \left(\frac{q(V - IR_s)}{\gamma k T_C} \right) - 1 \right] \quad (3)$$

The model proposed in (3) describes the working of a photovoltaic module under the hypothesis of knowing the values of γ , R_s , I_o and I_L .

These values are definable as function of the data usually specified into the datasheet of the modules' manufacturer

- $I_{SC,REF}$ (short-circuit current under standard conditions)
- $V_{OC,REF}$ (load-less voltage under standard conditions)
- $I_{MPP,REF}$ (maximum power point current under standard conditions)
- $V_{MPP,REF}$ (maximum power point voltage under standard conditions)

In order to take into account the variation of the diode saturation current and the photoelectric current when temperature and radiation change, with respect to standard conditions, the model is completed with the following equations:

$$I_O = I_{O,REF} \left(\frac{T_C}{T_{C,REF}} \right)^3 \exp \left[\left(\frac{q\epsilon_G}{k\gamma} \right) \left(\frac{1}{T_{C,REF}} - \frac{1}{T_C} \right) \right] \quad (4)$$

where:

$-\epsilon_G$ is the energy gap of the material with whom the cell is made (for the silicon it's 1,12 eV);

$$I_L = \left(\frac{G}{G_{REF}} \right) \cdot [I_{L,REF} + \mu_{ISC} (T_C - T_{C,REF})] \quad (5)$$

where:

$-G$ is the radiation [W/m^2]

$-G_{REF}$ is the radiation under standard conditions [W/m^2]

$-I_{L,REF}$ is the photoelectric current under standard conditions [[A]

$-T_{C,REF}$ is the module temperature under standard conditions [K]

$-\mu_{ISC}$ is the temperature coefficient of the short-circuit current [A/K], given by the manufacturer according to CEI EN 60891 standard.

The obtained equations allow to represent the I-V characteristic curve of one photovoltaic module under generic temperature and radiation conditions, when the characteristic parameters under standard conditions are known.

IV. MATHEMATICAL MODEL OF A PHOTOVOLTAIC MODULE IN MATLAB-SIMULINK ENVIRONMENT: IMPLEMENTATION AND VALIDATION

The mathematical model has been implemented in Matlab-Simulink environment; its validation has been carried out by comparing the characteristic curves, obtained from the simulations, with the curves given by the manufacturer.

Starting from the data sheet, using the equation given into the paragraph 3 and the equation which follows, which express the equation 3 with respect to the voltage, the mathematical model has been implemented and the block scheme is reported in Fig. 2.

$$V = \frac{\gamma k T_C}{q} \ln \left(\frac{I_L - I}{I_O} + 1 \right) - I R_S \quad (6)$$

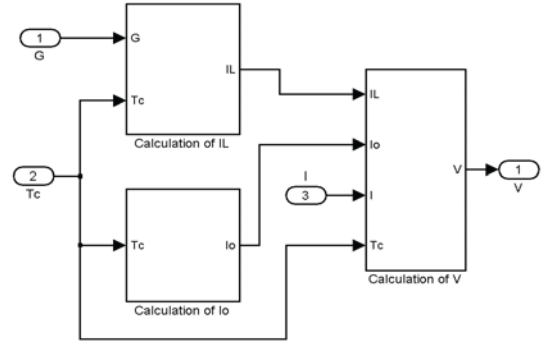


Fig. 2 Block scheme of the model of the photovoltaic module implemented.

The program allows to modify the environmental data and the characteristic parameters of the implemented photovoltaic module (e.g. temperature, radiation, temperature coefficient of the load-less voltage, short-circuit current, etc). In order to validate the model of a photovoltaic module, implemented in Matlab-Simulink environment, the characteristic parameters of one BP Solar multicrystal silicon BP SX 10 PV have been used.

Fig. 3 reports the characteristic I-V curves, given by the manufacturer, of the module under standard radiation condition ($G=1000 W/m^2$) with four various temperature values ($T=0^\circ C, 25^\circ C, 50^\circ C, 75^\circ C$) [8]. The model validation has been carried out by comparing the above mentioned characteristic I-V curves with the ones obtained from the simulations under the same radiation and temperature conditions (Fig. 4).

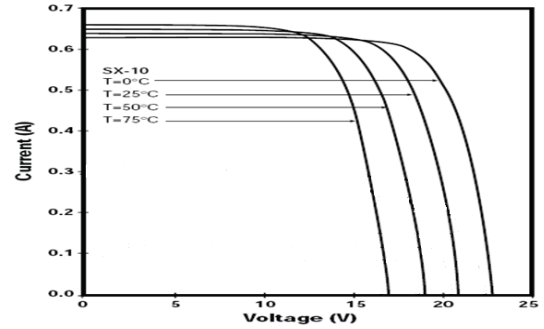


Fig. 3 SX 10 module characteristic I-V curves given by the BP Solar manufacturer.

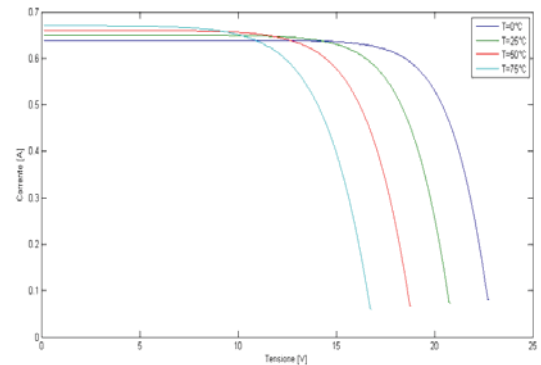


Fig. 4 Simulation I-V curves of the BP Solar SX 10 module ($G=1000W/m^2$).

By comparing the Fig. 3 and 4 it has been verified that in correspondence to the curves' knees, for the same voltage value, there is a maximum current value gap equal to 0.5% and so the obtained simulation results can be considered satisfactory.

V. MATHEMATICAL MODEL OF A PHOTOVOLTAIC (PV) FIELD EQUALLY OR UNEQUALLY RADIATED AND ITS IMPLEMENTATION IN MATLAB-SIMULINK ENVIRONMENT

In order to obtain the electric loads voltage and current values, several PV modules are series connected and eventually several PV strings are parallel connected.

So, in order to model a PV field starting from the model of a PV module, under the hypothesis of uniform environmental conditions of the field (radiance and temperature levels), the following equations can be used:

$$I_{L,TOT} = N_P I_L \quad (7)$$

$$I_{O,TOT} = N_P I_O \quad (8)$$

$$\gamma_{TOT} = N_S \gamma \quad (9)$$

$$R_{S,TOT} = \frac{N_S}{N_P} R_S \quad (10)$$

where N_p is the number of parallel modules and N_s the number of series modules.

The equations (7 ÷ 10) allow to obtain the mathematical model of a PV field, starting from the model of a PV module, under the hypothesis of uniform radiance and temperature levels.

Under the hypothesis of unequally radiance, on the contrary, it is not possible to simply take into account equations (7 ÷ 10).

Instead, it is needed to consider the voltage and current contributions that each module can supply thanks to its exposition conditions. Therefore the field model must take into account the series-parallel connection of modules which have different voltage and current outputs.

For this purpose, the equation (3) in terms of voltage is hereafter reported

$$V = \frac{\gamma k T_C}{q} \ln \left(\frac{I_L - I}{I_O} + 1 \right) - I R_S \quad (11)$$

Below, some simulation analyses related to the behaviour of a PV field made of four modules, devoid of bypass diodes, connected in several series-parallel configurations are presented.

The analysis is executed both in equally and unequally radiated conditions (one module of the PV field is supposed shaded) in order to evaluate which series-parallel configuration, under the same shading conditions, gives the biggest electrical power and so the lowest mismatch losses.

It has been considered a radiative value equal to $G=800 \text{ W/m}^2$ and a PV module temperature equal to $T=25^\circ\text{C}$.

The unequally radiative conditions have been simulated by imposing to the shaded panel a radiative

value equal to $G=500 \text{ W/m}^2$ with a PV modules temperature equal to $T=25^\circ\text{C}$.

The configurations taken into account are related to the following possible connections with 4 modules:

1. series connected modules under the same radiative conditions;
2. series connected modules with one shaded module;
3. parallel connected modules under the same radiative conditions;
4. parallel connected modules with one shaded module;
5. series connection of two parallel modules under the same radiative conditions;
6. series connection of two parallel modules with one shaded module;
7. two parallel PV strings, each consisting of two modules under the same radiative conditions;
8. two parallel PV strings, each consisting of two modules one of which is shaded;
9. two parallel PV modules series connected with two parallel PV modules under the same radiative conditions;
10. two parallel PV modules series connected with two parallel PV modules one of which is shaded;
11. two parallel PV modules, one of which is shaded, series connected with two other parallel PV modules;
12. three parallel PV modules series connected with one PV module under the same radiative conditions;
13. three parallel PV modules series connected with one shaded PV module;
14. three parallel PV modules, one of which is shaded, series connected with one PV module.

As an example, it is reported the model of the configuration with two parallel PV strings, each consisting of two modules one of which is shaded (Fig. 5) related to the simulation n. 8. Particularly, one of the two modules of one PV strings is shaded and so its radiance is equal to $G=500 \text{ W/m}^2$ whilst the other modules have G equal to 800 W/m^2 .

In Fig. 6 the implementation scheme used to carry out the simulations is reported.

In Tab. 1 the voltage, current and power output values, as the load changes, are reported.

In Fig. 7 and 8 the I-V and P-V curves, obtained with the examined configuration and compared with the curves obtained under the hypothesis of equally radiated field, are reported.

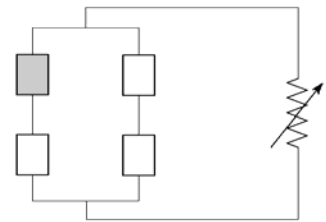


Fig. 5 – Circuital demonstrative scheme of the n. 8 experience simulation

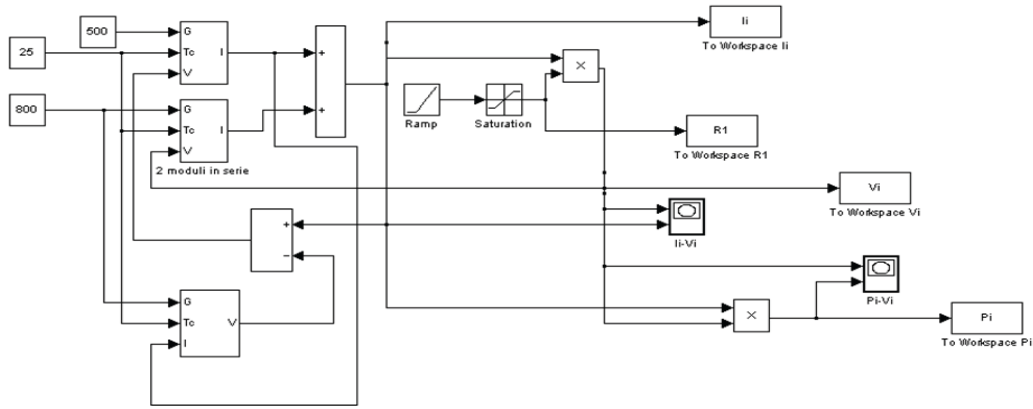


Fig. 6 – Implementation scheme related to the simulation n. 8

Tab.1 – Voltage, current and power values related to the simulation n. 8

V [V]	I [A]	P [W]
40,8	0,0272	1,1102
40,62	0,0694	2,818
39,46	0,2972	11,7268
37,94	0,51	19,354
35,16	0,7205	25,3343
33,38	0,78	26,0384
30,03	0,8249	24,7693
26,18	0,8393	21,9806
22,93	0,843	19,3307
14,87	0,8449	12,5624
10,14	0,845	8,5677
0,0008	0,845	0,0007

The power maximum value is equal to $P=26,04$ W with voltage and current values equal to $V=33,38$ V; $I=0,78$ A.

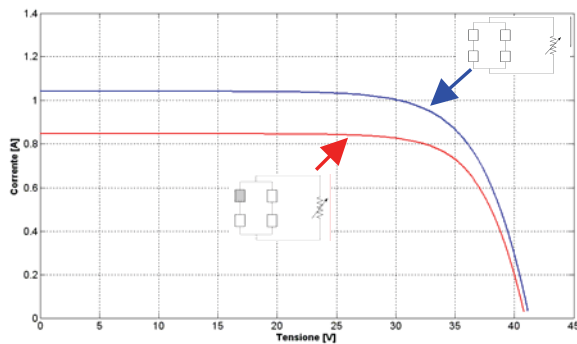


Fig. 7 - I-V curve related to the simulation 7 (blue);
I-V curve related to the simulation 8 (red).

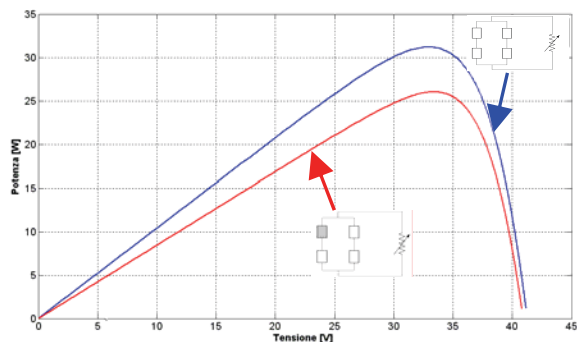


Fig. 8 - I-V curve related to the simulation 7 (blue);
I-V curve related to the simulation 8 (red).

VI. SIMULATION RESULTS

The simulation results related to the condition of equally radiancy (simulations 1, 3, 5, 7 and 9) show that the maximum power has always the same value, here equal to $P=31,20$ W. It was to be expected.

It means that, from the electrical energy production point of view, these connections can be used without any distinction. It must be underlined, however, that each configuration is characterized, in correspondence to the maximum power value equal to $P=31,20$ W, by several values of voltage and current.

The configuration with several modules series connected are to be preferred in order to reduce the currents.

Actually, it is needed to take into account many others factors which may influence the modules series and parallel placement like: the number and the size of the PV field inverters, the number of PV strings which may be parallel connected with an inverter; it must not be neglected, moreover, the effects connected to the real exposition conditions which may case unequally radiances, reason of mismatch losses. Moreover, the simulations related to the unequally radiancy in a four modules PV field have been carried out.

Particularly, a $G=800\text{W/m}^2$ value for the not shaded modules and a $G=500\text{W/m}^2$ value for the shaded module have been considered (corresponding to a dimming of 37,5%).

Taking into account the series connection of 4 modules, there is a reduction of the produced power of 28,3% with respect to the case of the equally radiated modules. Analogous comparison made for the case of parallel connection of the modules bring to a reduction of the produced power equal to 10%.

This first survey suggests that, under the condition of unequally radiancy, the power obtained from the plant is strongly influenced by its configuration. Therefore, besides the above mentioned connections, many series-parallel connections have been considered in order to identify the connection that gives the biggest output power in presence of shading.

The results obtained by the simulations are shown in Fig. 9 where it has also been highlighted the reference value of the power produced by the 4 modules (31,20 W) which does not feel the effect of the mismatch.

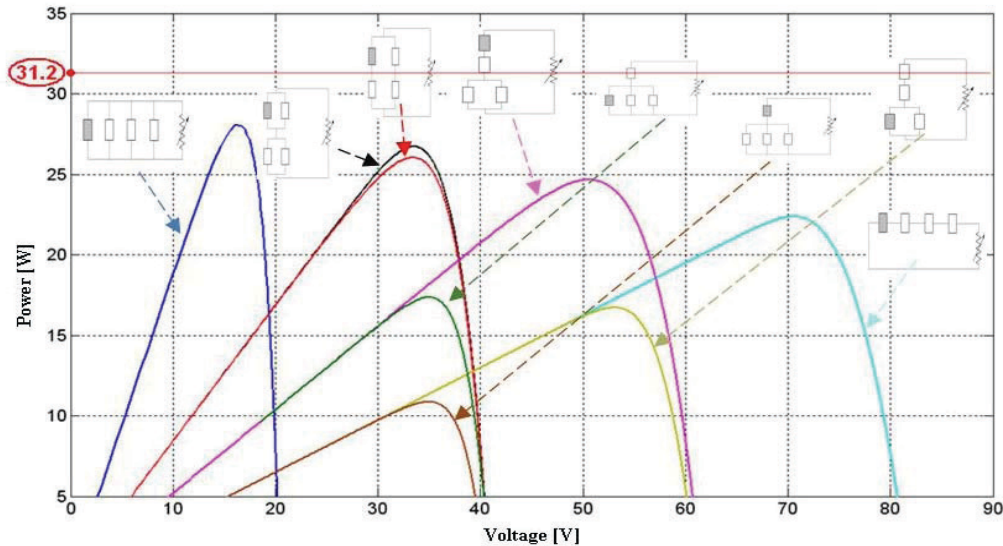


Fig. 9 - P-V curves obtained for the several configurations of one PV field made of 4 modules and unequally radiated.

From Fig. 9 it follows that the several examined configurations determine very different values of the peak powers and that the configuration that has the highest power is the one related to the simulation 4. Anyway in this configuration the current value could be too high and so the configuration to be preferred is the sixth. In this, the power losses with respect to the case of equally radiation condition is equally to 14,3%. In the worst case (configuration of simulation n. 13) the power losses, with respect to the case of equally radiancy, is equal to 65,2%.

The obtained results agree with the ones present in literature for analogous studies.

VII. CONCLUSIONS

In this paper a mathematical model of a PV module, implemented in Matlab-Simulink environment has been proposed.

This model has been validated by comparing the simulation results with the curves given by the manufacturers.

Moreover, the mathematical model of a PV field, both under equally and unequally radiation condition, has been here implemented.

The surveys and the simulations carried out allow to state that thanks to re-configuration techniques of the series-parallel connections of PV plants, under unequally radiation conditions, it is possible to optimize the production of electrical energy from PV plants.

This result opens scenarios of possible modifications of PV fields configurations which are today chosen during the design stage and are not changeable during operation conditions. It's clear that the possible re-configurations of the PV fields determine output values of the field voltages and currents which must be compatible with the inverter characteristics.

With this purpose it is particularly appropriate the usage of multilevel inverters connected to re-configurable fields.

Moreover, in the light of what above mentioned, further surveys can be represented by the research of new

algorithms which allow re-configurations of field connections in order to maximize the electrical power produced in PV plants.

REFERENCES

- [1] Z. M. Salameh, F. Dagher – “The effect of electrical array reconfiguration on the performance of a PV-powered volumetric water pump” IEEE Transactions on Energy Conversion, Vol. 5, No.4, December 1990
- [2] T. U. Townsend – “A method for estimating the long-term performance of direct-coupled photovoltaic system” MS Thesis, Solar Energy Laboratory, University of Wisconsin, Madison, (1989)
- [3] R. Chenni, M. Maklouf, T. Kerbache, A. Bouzid, - “A detailed modeling for photovoltaic cells”- Solar Energy 32, (2007).
- [4] J. H. Eckstein – “Detailed modeling of photovoltaic components” MS thesis, Solar Energy Laboratory, University of Wisconsin, Madison, (1999)
- [5] G. Walker – “Evaluating MPPT converter topologies using a Matlab PV model”
- [6] I. H. Altas, A. M. Sharaf, - “A photovoltaic array simulation model for Matlab-Simulink GUI environment” IEEE, (2007).
- [7] R. Candela, V. Di Dio, E. Riva Sanseverino, P. Romano – “Reconfiguration techniques of partial shaded PV system for the maximization of electrical Energy production” – IEEE-ICCEP 2007 International Conference on Clean Electrical Power, Capri, Italia, 21-23 May 2007
- [8] www.bpsolar.com

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